

Q-R-pre-preserving-conditional^{11,40}

$$\begin{aligned}
& \forall es:ES, P_1, P_2:(E \rightarrow \mathbb{P}), Q_1, R_1, Q_2, R_2:(E \rightarrow E \rightarrow \mathbb{P}), dcd_P_1:(e:E \rightarrow \text{Dec}(P_1(e))), \\
& \quad f_1:(\{e:E \mid P_1(e)\} \rightarrow E), f_2:(\{e:E \mid P_2(e)\} \rightarrow E). \\
& (\forall e, e':\{e:E \mid P_1(e)\} . \neg(Q_2(f_1(e), f_1(e')))) \\
& \Rightarrow (\forall e, e':\{e:E \mid P_2(e)\} . \neg(Q_1(f_2(e), f_2(e')))) \\
& \Rightarrow (\forall e:\{e:E \mid P_1(e)\} , e':\{e:E \mid P_2(e)\} . \\
& \quad (\neg(Q_1(f_1(e), f_2(e')))) \\
& \quad \& (\neg(Q_1(f_2(e'), f_1(e)))) \\
& \quad \& (\neg(Q_2(f_1(e), f_2(e')))) \\
& \quad \& (\neg(Q_2(f_2(e'), f_1(e'))))) \\
& \Rightarrow f_1 \text{ is } Q_1\text{-}R_1\text{-pre-preserving on } P_1 \\
& \Rightarrow f_2 \text{ is } Q_2\text{-}R_2\text{-pre-preserving on } P_2 \\
& \Rightarrow [P_1? f_1 : f_2] \text{ is } Q_1 \vee Q_2\text{-}R_1 \vee R_2\text{-pre-preserving on } P_1 \vee P_2
\end{aligned}$$